Acceleration of energetic particles by large-scale compressible magnetohydrodynamic turbulence

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ABSTRACT

Fast particles diffusing along magnetic field lines in a turbulent plasma can diffuse through and then return to the same eddy many times before the eddy is randomized in the turbulent flow. This leads to an enhancement of particle acceleration by large-scale compressible turbulence relative to previous estimates in which isotropic particle diffusion is assumed.

1. Introduction

Particle acceleration by magnetohydrodynamic (MHD) turbulence has been investigated by a number of authors (e.g., Fermi 1949, Hall & Sturrock 1967, Wentzel 1968, Kulsrud & Ferrari 1971, Eichler 1979, Achterberg 1981, Schlickeiser & Miller 1998), and has been studied in a number of astrophysical settings, including the interstellar medium (Ptuskin 1988, Schlickeiser & Miller 1998), solar flares (LaRosa et al 1994, Miller et al 1996, Miller et al 1997, Miller 1998, Schlickeiser & Miller 1998, Miller 1998), active galactic nuclei (Blackman 1999, Gruzinov & Quataert 1999), diffuse intracluster plasma in galaxy clusters (Eilek & Weatherall 1999, Weatherall & Eilek 1999, Dogiel 1999, Brunetti et al 1999, Ohno et al 2002, Fujita et al 2003), and the lobes of extragalactic radio sources (Eilek & Henriksen 1984, Manolakou et al 1999).

The way in which magnetohydrodynamic (MHD) turbulence accelerates energetic particles depends on the relative magnitudes of $\lambda_{\parallel}$ and $\lambda_{\rm mfp}$, where $\lambda_{\parallel}$ is the length of a turbulent eddy (or the wave length of a weakly damped wave) measured along the magnetic field, and $\lambda_{\rm mfp}$ is the energetic-particle scattering mean free path. In the weak-scattering limit, $\lambda_{\parallel} \ll \lambda_{\rm mfp}$, a particle's velocity $v_{\parallel}$ along the background magnetic field is approximately constant as the particle travels

many wave lengths along the magnetic field (assuming that the amplitudes of the fluctuations are sufficiently small), and the particle interacts resonantly with a weakly-damped wave when the wave frequency in the frame of reference moving along the magnetic field at speed $v_{\parallel}$ is an integer multiple of the particle's gyrofrequency. In the strong-scattering limit, $\lambda_{\parallel} \gg \lambda_{\rm mfp}$, a particle travels diffusively across an eddy of length $\lambda_{\parallel}$, and acceleration is non-resonant.

Particle acceleration in the strong-scattering limit is the focus of this paper. Ptuskin (1988) studied particle acceleration in the strong-scattering limit by a power-law spectrum of acoustic or magnetosonic waves extending from a large scale l to a smaller scale l_1, with rms fluctuation velocity $u_{\rm rms}$ dominated by the waves at scale l. He assumed that particle diffusion in space is isotropic with diffusion coefficient D, as opposed to primarily along field lines, and that some type of small-scale wave distinct from the acoustic/magnetosonic waves scatters the particles, with $\lambda_{\rm mfp} \sim D/v \ll l_1$, where v is the particle speed. He found that for $D \gg v_{\rm w} l$, where $v_{\rm w}$ is the wave phase velocity, particles diffuse in momentum space according to the equation

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_p \frac{\partial f}{\partial p}\right), \tag{1}$$

where f is the particle distribution function averaged over the waves, and the momentum diffusion coefficient D_p is given by

$$D_p = \frac{2p^2 u_{\rm rms}^2}{9D}, \tag{2}$$

where p is the particle momentum. The condition $D \gg v_{\rm w} l$ implies that particles diffuse a distance l in a time much shorter than the wave period at scale l. In this limit, the largest-scale waves make the dominant contribution to D_p.

This paper extends Ptuskin's analysis to strong compressible MHD turbulence taking into account the reversible motion of particles along magnetic field lines. It is assumed that the rms turbulent velocity $u_{\rm rms}$ equals or exceeds the Alfvén speed $v_{\rm A}$, and that there is a single dominant length scale (outer scale) denoted l for both velocity and magnetic fluctuations. Only the fluctuations at scale $\sim l$ are considered. As in Ptuskin's (1988) study, it is assumed that some type of small-scale wave scatters particles, and that $\lambda_{\rm mfp} \ll l$. It is assumed that eddies are randomized in a time $\tau_{\rm rand} \sim l/u_{\rm rms}$, and that $D_{\parallel} \gg u_{\rm rms} l$, where $D_{\parallel}$ is the diffusion coefficient for particle motion along field lines. Thus, a particle diffuses a distance greater than l during a time $\tau_{\rm rand}$, and can diffuse through and then return to an eddy multiple times before the eddy is randomized in the turbulent flow. It is shown in section 2 that this increases the coherence of stochastic particle acceleration, effectively increasing D_p above the value in equation (2) by a factor of $\sim N$, where $N \sim \sqrt{D_{\parallel}/ul}$ is the typical number of times a particle diffuses through and then returns to an eddy before the eddy is randomized, assuming a particle is tied to a single field line. The discussion of section 2 consists of estimates based on physical arguments; these estimates are tested and vali-

dated by Monte Carlo particle simulations in section 3. In section 4 the results of the paper are applied to particle acceleration in the interstellar medium and the lobes of radio galaxies.

It is known that small cross-field motions and the divergence of neighboring magnetic field lines in turbulent plasmas allow a particle to escape from its initial field line by traveling sufficiently far along the magnetic field. This places an upper limit on N, which is explained in the Appendix. It is shown that this upper limit is not important in the astrophysical examples considered in section 4 given the assumed values of $D_{\parallel}$. However, for sufficiently large values of $D_{\parallel}$, the upper limit on N becomes important.

2. Phenomenological description of momentum diffusion

Since the scattering mean free path is assumed to be $\ll l$ and only those eddies at scale $\sim l$ are considered, a particle's momentum p evolves according to the equation

$$\frac{dy}{dt} = -\frac{\nabla \cdot \mathbf{u}}{3}, \tag{3}$$

where

$$y = \ln(p/p_0), \tag{4}$$

p_0 is a particle's initial momentum, and $\mathbf{u}$ is the turbulent velocity associated with the large-scale eddies (Skilling 1975, Ptuskin 1988). As a particle diffuses in space, it encounters a series of eddies that cause random increments in y, leading to diffusion in y-space with diffusion coefficient

$$D_y = \frac{(\delta y)^2_{\rm rms}}{2\delta t}, \tag{5}$$

where δt is the time during which increments to y remain correlated, and $(\delta y)_{\rm rms}$ is the rms change in y during a time δt. It is assumed that the turbulent velocities are randomized in a time

$$\tau_{\rm rand} = \frac{bl}{u_{\rm rms}}, \tag{6}$$

where b is a constant of order unity. The characteristic time for a particle to diffuse through an eddy is

$$\tau_{\rm diff} = \frac{l^2}{2D_{\parallel}}. \tag{7}$$

It is assumed that $D_{\parallel} \gg u_{\rm rms} l$, and thus

$$\tau_{\rm diff} \ll \tau_{\rm rand}. \tag{8}$$

In this section, it is assumed that a particle remains tied to the same magnetic field line. Thus, given equation (8), a particle typically diffuses through an eddy and later returns to the same eddy before the eddy is randomized. During such successive encounters, the values of $\nabla \cdot \mathbf{u}$ within the eddy are correlated, and the corresponding increments to y are correlated. The correlation time δt for random steps in y-space is thus $\tau_{\rm rand}$, and not $\tau_{\rm diff}$.

The increment to y during a time $\tau_{\rm rand}$ is

$$\delta y = -\frac{1}{3}\int_0^{\tau_{\rm rand}} \nabla \cdot \mathbf{u} dt. \tag{9}$$

It is assumed that there is no average compression or decompression in the turbulent flow, so that

$$\langle \delta y \rangle = 0, \tag{10}$$

where $\langle \ldots \rangle$ indicates an average over an arbitrarily large number of random steps. During a time $\tau_{\rm rand}$, a particle's position is distributed over an interval of length $\sim Nl$, where

$$N = \frac{\sqrt{D_{\|}\tau_{\rm rand}}}{l}. \tag{11}$$

Since the values of $\nabla \cdot \mathbf{u}$ in different eddies are uncorrelated, the value of $\nabla \cdot \mathbf{u}$ averaged over a distance Nl is $\sim N^{-1/2}(\nabla \cdot \mathbf{u})_{\rm rms}$, where $(\nabla \cdot \mathbf{u})_{\rm rms}$ is the rms value of $\nabla \cdot \mathbf{u}$. Thus, the rms value of δy is approximately

$$(\delta y)_{\rm rms} \sim \frac{N^{-1/2}(\nabla \cdot \mathbf{u})_{\rm rms}\tau_{\rm rand}}{3}. \tag{12}$$

Since only eddies of scale l are considered, and since $u_{\rm rms} \gtrsim v_{\rm A}$,

$$(\nabla \cdot \mathbf{u})_{\rm rms} = \frac{a u_{\rm rms}}{l}, \tag{13}$$

where a is a constant of order unity. Thus, if $N \gg 1$, then $(\delta y)_{\rm rms} \ll 1$, and the probability $P(y,t)dy$ that $\ln(p/p_0) \in (y, y+dy)$ at time t satisfies the Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial^2}{\partial y^2}(D_y P), \tag{14}$$

where D_y, which is defined in equation (5), is given by

$$D_y \sim \frac{N a^2 u_{\rm rms}^2}{18 D_{\|}}. \tag{15}$$

The log-momentum diffusion coefficient D_y is analogous to Ptuskin's (1988) momentum diffusion coefficient $D_{p,\rm Ptuskin}$ divided by p^2. If Ptuskin's coefficient D for isotropic spatial diffusion is set equal to $D_{\|}$, then in the large-N limit

$$\frac{p^2 D_y}{D_{p,\rm Ptuskin}} \sim N. \tag{16}$$

This ratio can be understood as follows. Ptuskin's result is approximately recovered by taking $\delta t \sim \tau_{\rm diff}$. Such a relation is valid if spatial diffusion is isotropic, since there is only a small chance that a particle returns to the same eddy multiple times before the eddy is randomized when $D \gg l u_{\rm rms}$. However, when a particle diffuses along a single field line, the particle's location during a time $\tau_{\rm rand} = N^2 \tau_{\rm diff}$ is distributed among roughly N eddies, and thus a particle spends a time $\sim N\tau_{\rm diff}$ in an eddy before the eddy is randomized. The particle's random walk in y-space can thus be thought of consisting of individual steps of duration $N\tau_{\rm diff}$ and magnitude $\delta y = N\tau_{\rm diff}(\nabla\cdot\mathbf{u})_{\rm rms}/3$. The resulting value of D_y is a factor of N larger than in the isotropic-spatial-diffusion case in which a particle takes individual random steps in y-space of duration $\tau_{\rm diff}$ and magnitude $\delta y = \tau_{\rm diff}(\nabla\cdot\mathbf{u})_{\rm rms}/3$. Thus, the reversible motion of a particle along a field line makes the particle's random walk in y-space more coherent, and its diffusion in y-space more rapid, than in the isotropic-spatial-diffusion case.

One consequence of equation (15) is that $D_y \rightarrow \infty$ as $\tau_{\rm rand} \rightarrow \infty$. The reason is that if $\nabla\cdot\mathbf{u}$ were constant in time along a field line, increments to y would remain correlated forever if a particle were tied to that field line, and the particle would undergo superdiffusion in y-space. This phenomenon can be described approximately as follows. During a time t, a particle's position is distributed over $\sim M$ eddies, where

$$M = \frac{\sqrt{D_\parallel t}}{l}. \tag{17}$$

Since the values of $\nabla\cdot\mathbf{u}$ within different eddies are uncorrelated, the typical value of $\nabla\cdot\mathbf{u}$ averaged over M eddies is $\sim M^{-1/2}(\nabla\cdot\mathbf{u})_{\rm rms}$. For large M, the rms value of y for a particle obeys the equation

$$\frac{dy_{\rm rms}}{dt} \sim \frac{M^{-1/2}(\nabla\cdot\mathbf{u})_{\rm rms}}{3} \propto t^{-1/4}. \tag{18}$$

Integrating and squaring the result, one obtains

$$y_{\rm rms}^2 \sim \frac{16 l (\nabla\cdot\mathbf{u})_{\rm rms}^2\, t^{3/2}}{81\sqrt{D_\parallel}}. \tag{19}$$

3. Numerical simulations

In this section, particle acceleration by compressible turbulence is investigated with the use of Monte Carlo particle simulations. Particles are assumed to follow a single field line. Distance along the field line is denoted by the coordinate x. The values of $\nabla\cdot\mathbf{u}$ at space-time grid points $(x,t) = (ml, n\tau_{\rm rand})$ are set to either $+\tau_{\rm rand}^{-1}$ or $-\tau_{\rm rand}^{-1}$ with equal probability, where m and n are integers. The value of $\nabla\cdot\mathbf{u}$ between grid points is obtained by linear interpolation in space and time, which gives $(\nabla\cdot\mathbf{u})_{\rm rms}\tau_{\rm rand} = 2/3$, or, equivalently, $ab = 2/3$, where a and b are defined in

equations (6) and (13). During each time step of duration δt, a particle's y-value is incremented by an amount $-\nabla \cdot \mathbf{u}\delta t/3$, and there is an equal probability that its x-coordinate is either increased or decreased by an amount $\sqrt{2D_{\parallel}\delta t}$.

In figure 1, D_y is plotted for different values of $D_{\parallel}$. To express the results in terms of $u_{\rm rms}$ and l, it is assumed that $a = 1$. The values of D_y in the numerical simulations, depicted with solid triangles in figure 1, are obtained by dividing the average of y^2 for 10^4 simulated particles by $2t$ at $t = 130\tau_{\rm rand}$. The solid line represents the value of D_y in equation (15). The dashed line represents the scaling $D_y \propto D_{\parallel}^{-1}$, analogous to Ptuskin's (1988) scaling $D_p \propto D^{-1}$ for isotropic spatial diffusion of particles.

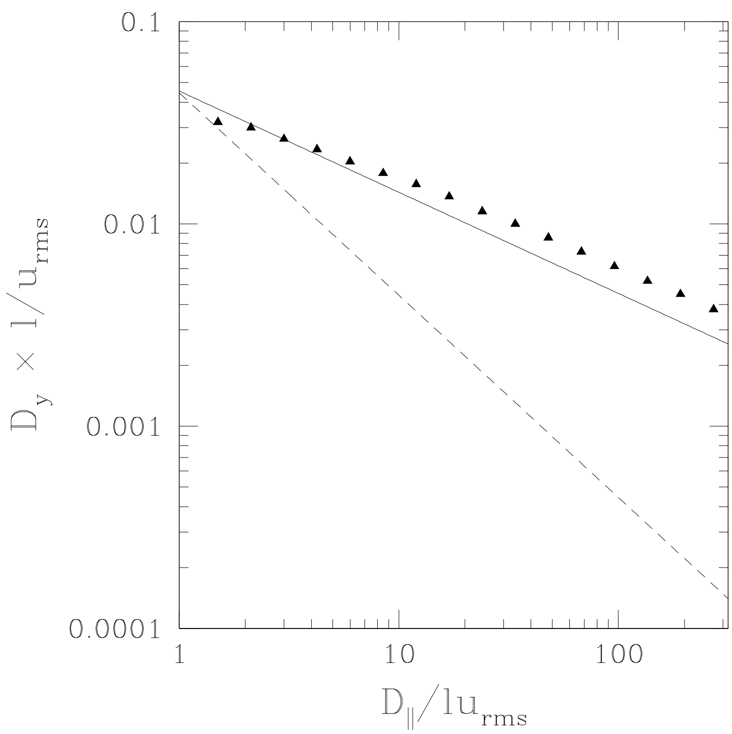

Fig. 1.— D_y as a function of $D_{\parallel}$. The solid triangles are the results of the Monte Carlo simulations. The solid line represents equation (15), and the dashed line represents a scaling of $D_y \propto D_{\parallel}^{-1}$.

The increase of $y^2_{\rm rms}$ with time when $\nabla \cdot \mathbf{u}$ is constant in time along a field line is shown in figure 2. The solid line is the value of $y^2_{\rm rms}$ in equation (19). The averages of y^2 for 10^4 simulated particles are given by the solid triangles and show superdiffusive behavior in approximate agreement with equation (19).

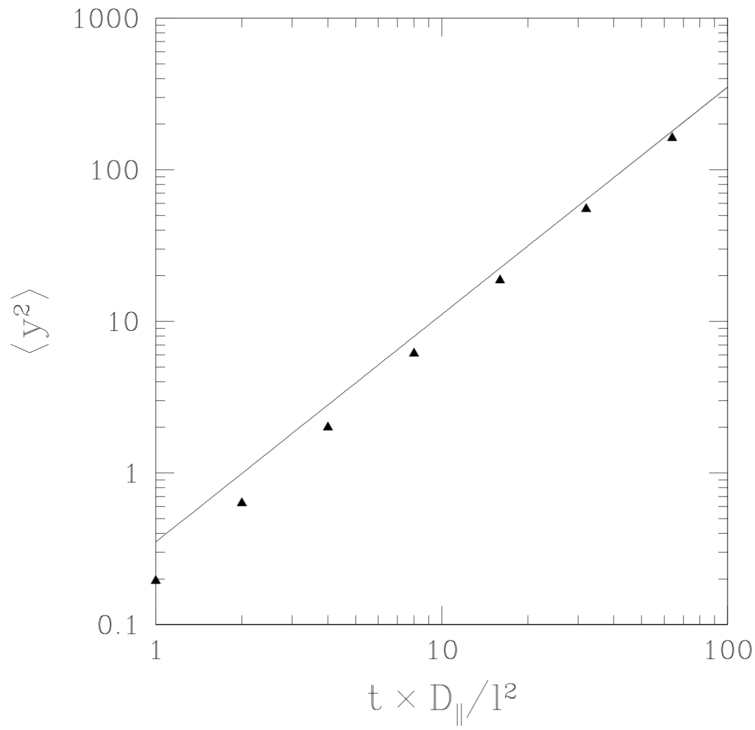

Fig. 2.— Superdiffusion in log-momentum-space when $\nabla \cdot \mathbf{u}$ is static along a field line. The solid triangles give the average of y^2 for 10^4 simulated particles that are tied to a single magnetic field line. The solid line is a plot of equation (19), in which $y_{\rm rms}^2 \propto t^{3/2}$.

4. Application to Galactic cosmic rays and relativistic electrons in radio galaxies

From equations (14) and (15) the characteristic time for particle acceleration by large-scale compressible turbulence is

$$\tau_{\rm acc} = D_y^{-1} \sim \frac{18 D_{\|}^{1/2} l^{1/2}}{u_{\rm rms}^{3/2}}, \tag{20}$$

where it is assumed that $a = 1$ in equation (13) and $b = 1$ in equation (6). The values of $D_{\|}$ for Galactic cosmic rays and relativistic electrons in the lobes of radio galaxies are not well known. There is disagreement as to the correct way to model the waves that cause energetic particle scattering (see, e.g., Bieber et al 1994, Schlickeiser & Miller 1998, Chandran 2000, Felice & Kulsrud 2001, Yan & Lazarian 2002). Some studies indicate that D and $D_{\|}$ have different values in different phases of the ISM (e.g., Felice & Kulsrud 2001), and the values of D and $D_{\|}$ may be different in the disk and halo of the Galaxy (Berezinskii et al 1990). In the hot coronal plasma of the

ISM, $u_{\rm rms} \sim 10^7$ cm/s (Shelton et al 2001). This value is comparable to the sound speed, and thus the turbulence is compressible. It is assumed that $l \sim 3 \times 10^{20}$ cm. If $D_{\parallel} = 10^{28}$ cm^2/s, then $\tau_{\rm acc} = 3 \times 10^7$ yr. If $D_{\parallel} = 10^{29}$ cm^2/s, then $\tau_{\rm acc} = 10^8$ yr. These values of $\tau_{\rm acc}$ are comparable to the Galactic confinement time of 1-GeV cosmic rays, and are the types of time scales required if reacceleration is to explain the peaking of secondary/primary ratios at a few GeV (Ptuskin 2001). For relativistic electrons in radio lobes, it is simply assumed that $D_{\parallel} = v_{\rm A} l_{\rm lobe}$, where $l_{\rm lobe} = 100$ kpc is the typical size of a radio lobe. For this value of $D_{\parallel}$, the time for electrons to diffuse out of a radio lobe is of order the Alfvén crossing time of the lobe. The thermal plasma density is taken to be 5×10^{-5} cm^{-3}, which is an an upper limit based upon Faraday depolarization, and B is taken to be 20 μG, a typical equipartition field (Spangler & Pogge 1984, Spangler & Sakurai 1985). These values yield $v_{\rm A} = 6 \times 10^8$ cm/s, and it is assumed that $u_{\rm rms} = v_{\rm A}$. The value of l for radio-lobe turbulence is taken to be $l_{\rm lobe}/30$ based on the standard deviation of the linear Stokes parameters in 3C 166 (Spangler 1983). For these parameters, $\tau_{\rm acc} \sim 5 \times 10^7$ yr. This type of time scale is comparable to the ages of relativistic-electron populations obtained from synchrotron aging estimates, which suggests that reacceleration may have to be taken into account in models of radio-lobe electrons.

5. Summary

It is shown that the reversible motion of energetic particles along the magnetic field enhances the rate of particle acceleration by large-scale compressible turbulence relative to the isotropic-spatial-diffusion case. When this effect is taken into account, the time scale for acceleration of relativistic electrons by large-scale compressible turbulence in the lobes of radio galaxies is $\sim 5 \times 10^7$ yr, and the time scale for acceleration of 1-GeV protons by large-scale compressible turbulence in the coronal plasma of the ISM is $\sim 3 - 10 \times 10^7$ yr. The main source of uncertainty in these estimates is the uncertainty in $D_{\parallel}$; other sources include the uncertainty in l, $u_{\rm rms}$, and the dimensionless constants a and b that relate $\tau_{\rm rand}$ and $(\nabla \cdot \mathbf{u})_{\rm rms}$ to $u_{\rm rms}$ and l.

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A. Particle escape from field lines through cross-field motion and field-line divergence

Particles with gyroradii ρ_g much less than the dominant magnetic field scale length l drift slowly across magnetic field lines because of gradients in the magnetic-field strength, curvature

of field lines, and wave pitch-angle scattering. Cross-field motions enhanced by the divergence of neighboring field lines allow particles to "escape" from their initial field lines, that is, to separate from their initial field lines by a distance l (Rechester & Rosenbluth 1978, Chuvilgin & Ptuskin 1996, Chandran & Cowley 1998). This can be seen with the aid of figure 3. Suppose a particle

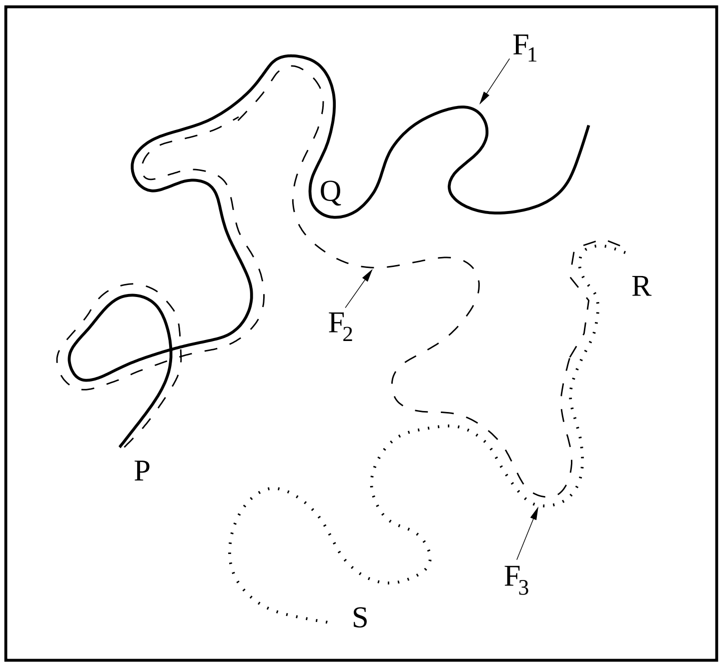

Fig. 3.— Diagram showing the way in which the divergence of neighboring field lines allows a particle to escape from its initial field line.

starts out at point P on field line F_1, and that its diffusive motion along the field initially moves it towards point Q. After moving a short distance, field gradients and collisions cause the particle to take a step of length $\sim \rho_g$ across the magnetic field, from F_1 to a new field line F_2. Although the particle continuously drifts across the field to new field lines, let us assume for the moment that it remains attached to F_2. As the particle diffuses along F_2, F_2 diverges from F_1. Let z_s be the distance that F_2 must be followed before F_2 separates from F_1 by a distance l. (Because the particle continuously drifts across the field, it typically separates from F_1 after traveling a distance somewhat less than z_s along the field; this effect is ignored in this discussion.) After the particle moves a distance z_s along F_2, its subsequent motion is not correlated with F_1. The particle proceeds to point R, and then its diffusive motion along the magnetic field changes direction, bringing it back towards point Q. Instead of following F_2 back to point Q, however, the particle drifts across

the field and ends up on a new field line F_3. After following F_3 for a distance $\sim z_s$, the particle separates from F_2 by a distance $\sim l$ and proceeds to point S. In this example, after the particle moves significantly farther than $\sim z_s$ along the magnetic field, it has a negligible probability of returning to within a distance $\sim l$ of its initial location. The maximum time a particle can remain correlated with an eddy is thus of order

$$\tau_{esc} = \frac{\langle z_s \rangle^2}{D_\parallel}, \tag{A1}$$

where $\langle z_s \rangle$ is the average value of z_s over a large sample of field-line pairs.

Several studies have calculated $\langle z_s \rangle$. Jokipii (1973) and Skilling, McIvor, & Holmes (1974) found that $\langle z_s \rangle \sim l$ for isotropic MHD turbulence with $\delta B \gtrsim B_0$, where δB is the fluctuating magnetic field and B_0 is the strength of any background field in the system that is coherent over distances $\gg l$. Narayan & Medvedev (2001) found $\langle z_s \rangle \sim l$ for locally anisotropic MHD turbulence with $\delta B \sim B_0$. A related quantity, the Kolmogorov entropy, which describes the exponential divergence of neighboring field lines while their separation is smaller than the dissipation scale of the turbulence, has been calculated by several authors (e.g., Rechester & Rosenbluth 1979, Zimbardo et al 1995, Barghouty & Jokipii 1996, Casse, Lemoine, & Pelletier 2001). Another related quantity, the diffusion coefficient for the wandering of single magnetic field line, has been studied by a number of authors (e.g., Matthaeus et al 1995, Michalek & Ostrowski 1998).

Maron, Chandran, & Blackman (2003) and Chandran & Maron (2003) calculated $\langle z_s \rangle$ by tracking field lines in direct numerical simulations of incompressible MHD turbulence with an inertial range extending from a dominant length scale l to a much smaller dissipation scale l_d. In simulations with $B_0 = 0$, they found that $\langle z_s \rangle$ asymptotes to a value of order several l as ρ_g is decreased towards the dissipation scale l_d in the large-l/l_d limit. From figures 5 and 7 of Chandran & Maron (2003), this value is $\sim 5-7l$. We assume that $\langle z_s \rangle$ has a similar value in turbulence with a mean magnetic field provided $\delta B \gtrsim B_0$.

The escape of particles from field lines modifies the estimate of D_y presented in section 2 when $\sqrt{D_\parallel \tau_{rand}} > \langle z_s \rangle$. In this case, a particle remains correlated with a magnetic field line for a time τ_{esc} that is less than τ_{rand}. During the time τ_{esc} the particle travels a distance $\sim \langle z_s \rangle$, and its location is spread out over roughly

$$N' = \frac{\langle z_s \rangle}{l} \tag{A2}$$

eddies. The arguments leading to equations (14) and (15) can be repeated, replacing τ_{rand} with τ_{esc}, to obtain

$$D_y \sim \frac{N' a^2 u_{rms}^2}{18 D_\parallel}. \tag{A3}$$

To summarize, if $\tau_{esc} < \tau_{rand}$, then equation (A3) is valid. If $\tau_{esc} > \tau_{rand}$, then equation (15) is valid.

For the astrophysical examples considered in section 4, $\tau_{esc} \geq \tau_{rand}$, and thus equation (15) can be applied. However, for larger values of $D_{\|}$, τ_{esc} can be reduced below τ_{rand}, in which case cross-field motion becomes important and equation (A3) applies.

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